Session 2 Solutions

Sol. 1 →

(a) $\lim_{x \to 5} x^2 + 2x + 3 = 5^2 + 2(5) + 3 = 38$

(b) $\lim_{x \to 0} \frac{x^2 + x}{x} = \lim_{x \to 0} \frac{\cancel{x}(x+1)}{\cancel{x}} = 0 + 1 = 1$

(c) $\lim_{x \to 2} \frac{7x - 14}{x^2 - 4x + 4} = \lim_{x \to 2} \frac{7\cancel{(x-2)}}{(x-2)^{\cancel{2}}} = \lim_{x \to 2} \frac{7}{x-2} = \infty$

(d) $\lim_{x \to -3} \frac{6x+18}{(x+4)(x+3)} = \lim_{x \to -3} \frac{6\cancel{(x+3)}}{(x+4)\cancel{(x+3)}} = \frac{6}{-3+4} = 6$

Sol. 2 →

$\lim_{x \to 2} f(x) = \lim_{x \to 2} x - 2 = 2 - 2 = 0$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x = 0$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x - 2 = -2$

Since $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

$\therefore \lim_{x \to 0} f(x)$ does not exist

Sol. 3 → (a) $\frac{df}{dx} = \frac{d}{dx}(x^2 + 2x + 4) = \frac{d}{dx}(x^2) + \frac{d}{dx}(2x) + \frac{d}{dx}(4)$

$= 2x + 2\frac{d(x)}{dx} + 0$

$= 2x + 2$

(b) $\frac{df}{dx} = \frac{d}{dx}(x^3) = 3x^2$

Sol. 4 →

$$\frac{\partial f}{\partial x} = 3x^2 + 4x + 6xy + 3y$$

$$\frac{\partial f}{\partial y} = 3x^2 + 3x$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} 3x^2 + 4x + 6xy + 3y \\ 3x^2 + 3x \end{bmatrix}$$

Sol. 5 →

$$\frac{\partial f}{\partial x} = 1 + z^2$$

$$\frac{\partial f}{\partial y} = 1$$

$$\frac{\partial f}{\partial z} = 2zx$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix} = \begin{bmatrix} 1 + z^2 \\ 1 \\ 2zx \end{bmatrix}$$

Sol. 6 →

$$\frac{d\sigma}{dx} = \lim_{h \to 0} \frac{\sigma(x+h) - \sigma(x)}{h} = \lim_{h \to 0} \left(\frac{e^{x+h}}{1 + e^{x+h}} - \frac{e^x}{1 + e^x} \right) \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{e^{x+h}(1 + e^x) - e^x(1 + e^{x+h})}{(1 + e^{x+h})(1 + e^x)\,h}$$

$$= \lim_{h \to 0} \frac{e^{x+h} + \cancel{e^{2x+h}} - e^x - \cancel{e^{2x+h}}}{(1 + e^{x+h})(1 + e^x)\,h}$$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^{x}}{(1+e^{x+h})(1+e^{x})h}$$

$$= \lim_{h \to 0} \frac{e^{x}(e^{h}-1)}{(1+e^{x+h})(1+e^{x})h}$$

$$= \frac{e^{x}}{(1+e^{x})^2} \cdot 1 \quad \left(\because \lim_{h \to 0} \frac{e^{h}-1}{h} = 1 \text{ is given}\right)$$

$$= \frac{e^{x}}{(1+e^{x})^2}$$

<u>Sol. 7</u> → $\frac{d}{dx}\left(\frac{x^3+3}{x^5+2x^2}\right) = \frac{(x^5+2x^2)(3x^2) - (x^3+3)(5x^4+4x)}{(x^5+2x^2)^2}$

$$= \frac{-2x^7 - 13x^4 - 12x}{(x^5+2x^2)^2}$$